"Barrier-Reef" Model of Resonance Particles*

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A number of resonance particles are described in terms of a potential-well model of two-particle systems. It is found that the systems may have resonance states in the continuum and may be held back from decaying by a centrifugal barrier reef. The short, calculated, lifetimes are consistent with those observed for the resonance particles. The variation of the potential-well depths with quantum numbers of the resonance particles is also discussed.

INTRODUCTION

THE intent of this investigation has been primarily to find out whether the lifetimes of resonance particles can be understood in terms of the decay of a resonance level of constituent interacting particles. Only two-particle systems of resonance particles have been considered. Since the nature of the potential between two "elementary" particles is generally unknown, this investigation is very preliminary. As a starting point, the simplest assumption of a spherical square-well potential is made. Further, it is assumed that the width of the potential well is equal to the distance of separation of the constituent particles at the instant of decay of the resonance particle. That is, the relative separation a of the constituent particles is given by

$$a^2=l(l+1)\hbar^2/p^2, \quad (1)$$

where l is the angular momentum assignment of the resonance particle and p is the center of mass momentum of the product particles arising from the decay.

In Table I, the $l=1$ resonance particles of the $1-$ boson octet and the $\frac{3}{2}+$ baryon decuplet are given, together with their product-, and therefore presumed constituent-, particle pairs. From the observed mass M of the resonance particles and the energy releases W to their decay products, the c.m. momenta p have been directly computed. The ranges a have then been obtained using Eq. (1). (Although the interpretation of what a is depends on the assumptions of the model, the value of a is model independent.)

TABLE I. $l=1$ resonance particles.

Vector bosons	M (MeV)	W (MeV)	p (MeV/c)	a (F)	V_0 (MeV)
$\rho(2\pi)$	757	477	351	0.80	573
$K^*(K\pi)$	888	254	285	0.98	560
$\varphi(2K)$	1020	32	126	2.22	195
$\frac{3}{2}+$ Baryons					
$N^*(N\pi)$	1237	159	231	1.21	470
$\Lambda^*(\Lambda\pi)$	1385	130	207	1.35	420
$\Xi^*(\Xi\pi)$	1533	72	148	1.89	294

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It is clear from Table I that the energies of the constituent particles are generally so high that the problem must be treated relativistically. Since we are concerned mainly with spinless products, π's and K's, we will describe the systems by the relativistic Schrödinger equation. In the region inside the potential well, if the depth is represented by a scalar potential $-V_0$, the radial equation is

$$(1/r^2)(d/dr)(r^2dR_i/dr)+\{(E+mc^2+V_0)^2-m^2c^4\} \times(R_i)(1/\hbar^2c^2)-\{l(l+1)\}(R_i)(1/r^2)=0, \quad (2)$$

where E is the energy of the system and m is the reduced mass of the constituent particles. In the region outside the well, the wave equation is

$$(1/r^2)(d/dr)(r^2dR_0/dr)+\{W^2+2Wmc^2\} \times(R_0)(1/\hbar^2c^2)-\{l(l+1)\}(R_0)(1/r^2)=0, \quad (3)$$

where W is the sum of the observed kinetic energies of the decay particles in the c.m. system.

For the particles of Table I, it is found that the centrifugal barrier term at $r=a$ is less than the other term in the brackets, and the wave function is oscillatory on both sides of the well boundary. A diagram showing the relative magnitudes of the three energy terms, V_0, $l(l+1)\hbar^2/2mr^2$, and $(W^2+2Wmc^2)/2mc^2$, in the case of the φ resonance particle, whose energies are the least relativistic, is given in Fig. 1. Generally, the centrifugal barrier is so high in relation to the energies of the free particles, and it will be shown that the bottom of the well is so deep, that strong partial reflections occur when particles attempt to escape from the well. Consequently, for certain energies, particles are trapped within the well for significantly longer times than it takes the particles to merely traverse distances of the width of the well.

RESONANCE-LEVEL WIDTHS

The energy E of the particles inside the well is complex and is given, in the usual fashion of α-particle decay, by the expression

$$E=W-\tfrac{1}{2}i\hbar\lambda, \quad (4)$$

where λ is the decay probability per unit time. Although all values of E are possible in the continuum of the

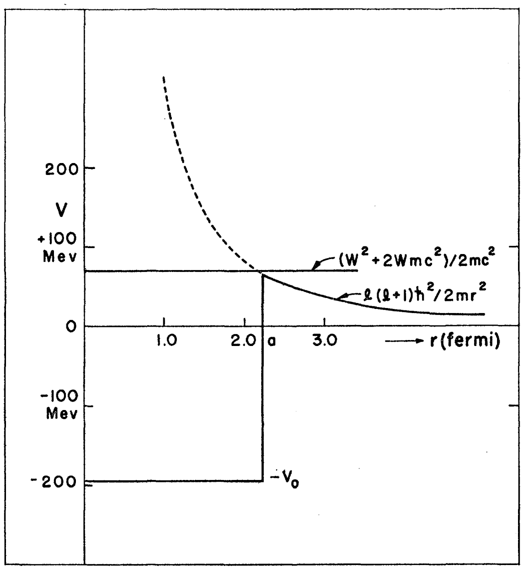

FIG. 1. Effective energy of constituent particles, in the case of the φ resonance particle, compared with the barrier reef potential.

system, it is very probable that the resonance-particle states are to be identified with energy regions of comparatively low values of λ. The relationship of λ to E can be found by matching the real and imaginary parts of the logarithmic derivatives of R_i and R_0 on opposite sides of the boundary at $r=a$. For the $l=1$ states, the forms of the radial solutions of (2) and (3) are

$$R_i \sim j_1(\rho_i), \quad (5)$$

where $\rho_i=\alpha r$ and

$$\alpha=[\{W+mc^2+V_0-\tfrac{1}{2}(i\hbar\lambda)\}^2-m^2c^4]^{1/2}(1/\hbar c),$$

and

$$R_0\sim i\{j_1(\rho_0)+in_1(\rho_0)\}, \quad (6)$$

where $\rho_0=\beta r$ and $\beta=[(1/\hbar^2c^2)(W^2+2Wmc^2)]^{1/2}$. The matching condition then gives

$$(\alpha a)^2 \sin(\alpha a)/\{\sin(\alpha a)-(\alpha a)\cos(\alpha a)\}=\delta(1+i\beta a), \quad (7)$$

where $\delta=\{1+(1/\beta^2a^2)\}^{-1}$. The solution of Eq. (7) need only be obtained for the imaginary part. If $\hbar\lambda \ll (W+mc^2+V_0)$, a condition which is certainly the case in the vicinity of the resonance, the width $\hbar\lambda$ can be obtained from the following form of Eq. (7), viz:

$$g=\delta\beta a(\sin f-f\cos f)/(\delta f\sin f-2f\sin f-f^2\cos f), \quad (8)$$

where $\alpha a=f+ig$, $f=\xi_0(a/\hbar c)$, $g=(1/2)(\hbar\lambda)(a/\hbar c)$,

$$\xi_0{}^2=\xi^2-m^2c^4, \quad \text{and} \quad \xi=W+mc^2+V_0. \quad (9)$$

Equation (8) has its first zero value when $f=4.494$. This value of f also corresponds to R_i being equal to zero at the boundary $r=a$ and to the condition that there is no imaginary part of E. The values of $|g|$ at neighboring points of f have been calculated from (8) and are shown for the $l=1$ particles in Fig. 2.

In the absence of partial reflections, it is clear that the lifetimes of the resonance particles should be $\sim a/c$. The range of energies over which the decay particles might sample the decay probabilities, which are proportional to $|g|$, are therefore $\sim\hbar c/a$ or ξ_0/f. Average values of $|g|$ have been obtained for the $l=1$ particles from the curves of Fig. 2 and these values of $|\bar{g}|$, and the calculated $\hbar\lambda$ widths, are given in Table II. These widths are in very reasonable accord with the experimental values, also listed in Table II. We conclude therefore that the model is capable of accounting for the observed lifetimes of the resonance particles, considered as states of the continuum of two-particle systems.

TABLE II. Widths of $l=1$ resonance particles.

Resonance particle	ξ (MeV)	$\lvert\bar{g}\rvert$	$\hbar\lambda_{\rm cal}$ (MeV)	$\hbar\lambda_{\rm obs}$ (MeV)
$\rho(2\pi)$	1110	0.17	85	20–120
$K^*(K\pi)$	906	0.09	37	50 ± 10
$\varphi(2K)$	468	0.05	9	~3
$N^*(N\pi)$	740	0.09	30	90 ± 20
$\Lambda^*(\Lambda\pi)$	663	0.09	26	50 ± 10
$\Xi^*(\Xi\pi)$	485	0.07	15	~7

WELL DEPTHS

Since the $l=1$ resonance particles, on this model, are identified with the level at f equal to 4.494, the values of the well depths V_0 may be obtained from the observed values of W, a, and the following equation based on (9):

$$(W+mc^2+V_0)^2-m^2c^4=f^2\hbar^2c^2/a^2, \quad (10)$$

where $f=4.494$, for the $l=1$ particles. The values of V_0 calculated from the appropriate values of a and W in Table I are given in the final column of Table I.

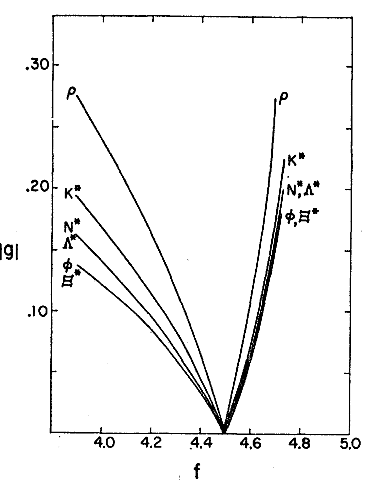

FIG. 2. Variation of decay probability $\sim|g|$ with continuum energy $\sim f$ for the case of the $l=1$ resonance particles. The imaginary and real quantities g and f, respectively, are defined by Eq. (9) in the text.

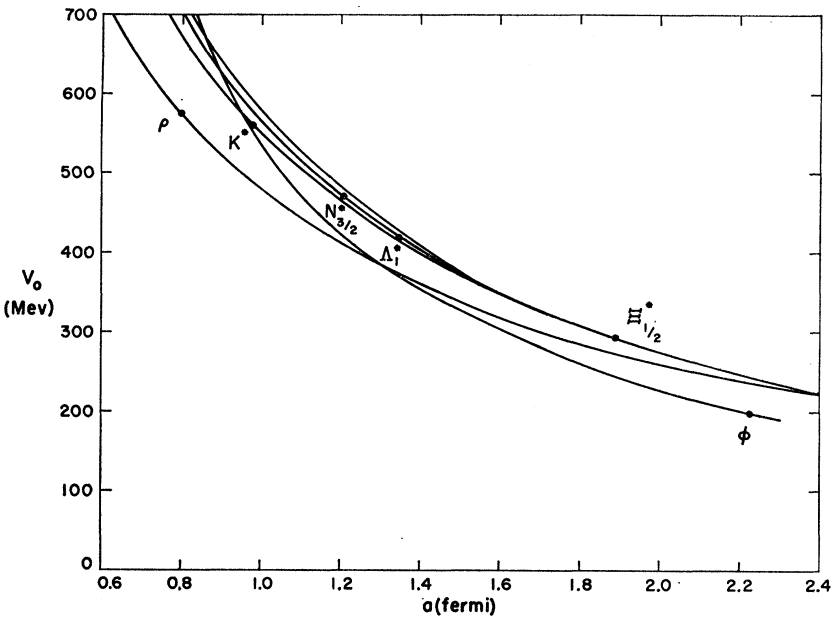

FIG. 3. Variation of well depth V_0 with range a for the $l=1$ resonance particles. Each resonance particle lies on its own curve at a point determined by its characteristic value of a, which is obtained from Eq. (1).

TABLE III. $l=2$ resonance particles.

$\frac{3}{2}-$ Baryons	M (MeV)	W (MeV)	p (MeV/c)	a (F)	V_0 (MeV)
$N^*(N\pi)$	1517	439	452	1.07	527
$Y_0^*(NK)$	1520	88	243	1.99	253
$Y_0^*(\Sigma\pi)$	1520	186	263	1.84	328
$Y_1^*(NK)$	1660	228	404	1.20	463
$Y_1^*(\Lambda\pi)$	1660	405	440	1.10	528
$Y_1^*(\Sigma\pi)$	1660	326	404	1.27	467

Because W is related to the momenta p of the constituent particles, and p is related to a by Eq. (1), V_0 can be expressed as a function of a. Curves showing the variation of V_0 with a for the $l=1$ resonance particles are given in Fig. 3. The locations of the resonance points, which are determined by the observed values of a in accordance with Eq. (10), are shown on the curves.

We now investigate the higher angular-momentum resonances of the same constituent particles. The $l=2$ resonance particles of the 3/2− baryon octet are listed in Table III. The $l=2$ bosons are omitted because they do not appear to be as well determined as the baryons. However, even for the $l=2$ baryons, identification of the constituent pairs is not as straight forward as for the $l=1$ baryons. This is because there are a number of alternative decays and, sometimes, even three-particle decays. The two-particle decays, which are the dominant ones, are given in Table III. One feature of the alternative decays is worth emphasizing. Even though the energy releases in alternative decays may be quite different, the c.m. momenta (and therefore consequently the computed relative constituent-particle separations) are rather closely the same. This seems to imply that the alternative constituent-pairs represent mixing or overlapping configurations of the same resonance system.

For the $l=2$ states of a spherical square-well potential, the radial wave function has the form:

$$R_i \sim (3/\rho_i^3)\sin\rho_i - (1/\rho_i)(\sin\rho_i) - (3/\rho_i^2)(\cos\rho_i). \quad (11)$$

As in the case of the $l=1$ particles, one might expect the $l=2$ resonances to occur when R_i is zero at $r=a$. This happens when ρ_i equals 5.77, and values of V_0 for the $l=2$ particles may be derived from an equation similar to (10) when the appropriate values of a and W from Table III are used. These values of V_0 are given in the final column of Table III. They are also plotted, together with the associated V_0 versus a curves in Fig. 4. It is seen that the $l=2$ particles occupy essentially the same regions of V_0 and a as the $l=1$ resonances.

PHENOMENOLOGICAL VARIATION OF V_0

Assuming that resonance particles may be described as resonance systems of constituent particles, it seems plausible that the strengths and ranges of the forces between the particles might be related to the quantum numbers of the system. A search has therefore been made for some phenomenological quantum number expression which correlates with either V_0 or a. (V_0 and a are not independent but are related by one of the curves of Fig. 3 or 4.)

On the basis of the nine resonance particles studied, the only clear correlation is that particles of larger isotopic spins have, other quantities being equal, higher values of V_0. For example, between identical constituent pairs, such as (KN) in the Y_0^* and Y_1^* resonances with

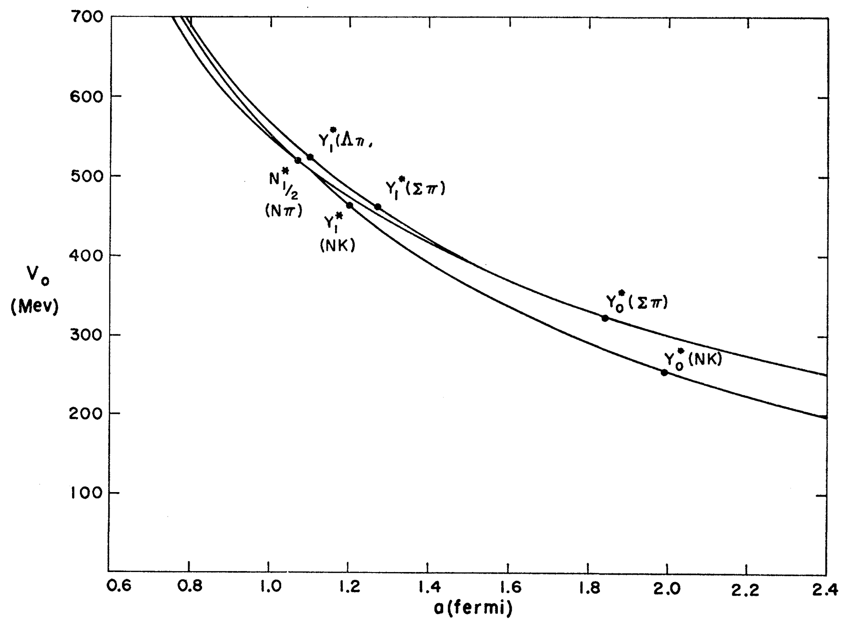

FIG. 4. Variation of well depth V_0 with range a for the $l=2$ baryon resonances. The location of a particular resonance particle is determined by the characteristic value of a derived from the observed energy of a particular pair of decay particles. Thus Y_0^* has two points corresponding to the alternative decays into NK and $\Sigma\pi$. The Y_1^* has three points corresponding to the alternative decays into NK, $\Lambda\pi$, and $\Sigma\pi$. One observes that the alternative points cluster together.

TABLE IV. Resonance-particle quantum numbers.

Particle	T	l	s	B	G	F	V_0 (ph) (MeV)	V_0 (mod) (MeV)
ρ	1	1	0	0	+1	3	680	573
K^*	$\frac{1}{2}$	1	+1	0		2	520	560
$N_{1/2}^*$	$\frac{1}{2}$	2	0	1		$\frac{3}{2}$	440	527
Y_1^*	1	2	−1	1		$\frac{3}{2}$	440	493
$N_{3/2}^*$	$\frac{3}{2}$	1	0	1		$\frac{3}{2}$	440	470
Λ^*	1	1	−1	1		$\frac{1}{2}$	280	420
Ξ^*	$\frac{1}{2}$	1	−2	1		$\frac{1}{2}$	280	294
Y_0^*	0	2	−1	1		$\frac{1}{2}$	280	294
φ	0	1	0	0	−1	0	200	195
f^0	0	2	0	0	+1	3		
Λ^0	0	0	−1	1		$\frac{3}{2}$		
Σ	1	0	−1	1		$\frac{1}{2}$		

$l=2$, the Y_1^* has the higher V_0. Similarly, for the $(\pi\Sigma)$ pairs in the Y_0^* and Y_1^* resonances, the Y_1^* has the higher V_0. There is also an indication from the variation of V_0 for the 3 resonances: ρ, K^*, and φ, all of which have the same l, but which are, of course, between different constituent particles, that higher isotopic spins have higher values of V_0.

A list of a number of quantum numbers for the nine resonance particles studied in this investigation is given in Table IV. A phenomenological number F equal to $|T+l+(1/2)s-B+G|$ has been found capable of ordering the nine particles according to their magnitudes of V_0. No underlying significance of this quantity is known, nor is it claimed unique, though it is the only one yet found. In order to make a test of whether this factor has any general significance, it has further been related to V_0 through the expression: $V_0=(200+160F)$ MeV. These values of $V_{0(\mathrm{ph})}$ are given in column 8 of Table IV and are compared with the potential-well model values given in column 9.

A possible test of the correlation may be afforded through the f^0 particle whose quantum numbers are: $T=0$, $l=2$, $s=0$, $B=0$, $G=+1$. These assignments yield $F=3$ and $V_0=680$ MeV. The value of a corresponding to this V_0 may be read off the ρ (or $\pi\pi$) curve of Fig. 3. The value is $a=0.65$ fermi. With these values of V_0 and a, the mass of the f^0 particle may be computed on the assumption of an $l=2$ resonance at $\rho_i=5.77$. The value of M so obtained is 1305 MeV, and the observed value is 1253 MeV. (If, instead of the value $V_0=680$ MeV, the ρ-particle value of $V_0=573$ MeV had been chosen, the calculated f^0 mass would have been 1080 MeV. The proper value of V_0 may lie between these estimates.)

Although there are as yet no other resonance particles of widely accepted isotopic spin and angular-momentum values, we have been tempted to examine the possibility that certain long-lived hyperons are also resonance-particle systems. (Such possibilities have already been considered from time to time, e.g., the Sakata and Goldhaber models, etc.) Specifically we may ask at this time whether it is possible to consider the Λ^0 as an s-state mixture of $P\bar{K}^-$ and $N\bar{K}^0$ systems, and to consider the Σ's as similar s-state pairs of: NK^0 and $P\bar{K}^-(\Sigma^0)$, $P\bar{K}^0(\Sigma^+)$, and $N\bar{K}^-(\Sigma^-)$.

The value of F for the Λ^0 is $\frac{3}{2}$ and the values of V_0 (based on the V_0 formula), and of a (based on the $N\bar{K}$ curve of Fig. 4) are 440 MeV and 1.26 F, respectively. For these values it is found that the lowest energy $l=0$ state is bound and occurs at $\rho_i=2.28$. The mass of the $N\bar{K}$ pair associated with this level is 1152 MeV. This is to be compared with the Λ^0 mass of 1115 MeV. The value of F for the Σ is $\frac{1}{2}$ and the correlated values of V_0 and a for an $N\bar{K}$ pair are 280 MeV and 1.85 F, respectively. For these values it is calculated that an $N\bar{K}$ pair has a lowest-energy $l=0$ bound state at $\rho_i=2.41$, and the corresponding mass of this system would be 1240 MeV. This is to be compared with the average mass of the Σ's equal to 1193 MeV. Since both the Λ^0 and Σ systems are considered as bound states of N and K, it is presumed that they would have lifetimes comparable to the decay times of the K particle themselves. Such would be the case if the K decays occur through the $K_1{}^0$ channel.

It is clear that the present model could lead to many resonances arising from many levels, not only from different l values but also from the same l. Clearly also the model has no bearing on those quantum numbers of the resonances which actually do exist. Given these quantum numbers, the values of V_0 and a, which this phenomenological model proposes, are purely conjectural.

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W-Meson Decays in Unitary Symmetry*

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A simple model for the axial vector current is proposed and generalized to SU_3. This model, together with the conserved vector current theory, is used to calculate several two-meson decay modes of the W meson.

IN searching for the W meson hypothesized to mediate the weak interactions, an approximate value for its decay rates into various channels is required. Here we present a calculation of several of its decay rates using a model for the weak interactions, incorporated in the theory developed by Cabibbo.[1]

The forms of the individual vector and axial vector currents entering the theory are obtained by considering only the pseudoscalar meson octet and the vector meson octet; interactions with baryons are not included. Strong interactions between the octets are taken into account by a phenomenological unitary symmetric coupling term.

First, let us consider the axial vector current and for the moment we shall discuss only the ρ and π mesons. The generalization to unitary symmetry will be made later.

In their derivation of the Goldberger-Treiman relation for the π meson, Gell-Mann and Lévy[2] assumed that the divergence of the axial vector current is proportional to the π-meson field. In the absence of strong interactions this leads to a weak current for the pion proportional to $\partial_\mu\pi$. The presence of a $\rho-\pi-\pi$ coupling can be described by introducing a phenomenological interaction Lagrangian[3]

$$\mathcal{L}_I=h\epsilon^{ijk}\rho_\mu{}^i\pi^j\partial_\mu\pi^k\,, \tag{1}$$

where i, j, and k are isotopic spin indices. The constant h can be determined from the width of the ρ. This leads to an equation of motion for the pion

$$(\Box+m_\pi{}^2)\pi^i=-2h\epsilon^{ijk}\rho_\mu{}^i\partial_\mu\pi^k. \tag{2}$$

So if we write an axial vector current that is proportional to

$$\partial_\mu\pi^i+2h\epsilon^{ijk}\rho_\mu{}^j\pi^k \tag{3}$$

then, with the restriction

$$\partial_\mu\rho_\mu=0\,, \tag{4}$$

the current is seen to have a divergence proportional to the pion field. The restriction (4) is, of course, valid only when we keep terms linear in h, consistent with the phenomenological nature of the model.

These considerations can be generalized to incorporate unitary symmetry by writing the total Lagrangian including the interaction term as

$$\mathcal{L}=-\tfrac{1}{4}F_{\mu\nu}{}^iF_{\mu\nu}{}^i+\tfrac{1}{2}M_i{}^2V_\mu{}^iV_\mu{}^i+\tfrac{1}{2}\partial_\mu P^i\partial_\mu P^i -\tfrac{1}{2}m_i{}^2P^iP^i+hf^{ijk}V_\mu{}^iP^j\partial_\mu P^k\,, \tag{5}$$

where P^i and V^i represent the pseudoscalar and vector meson octets, respectively, and $F_{\mu\nu}{}^i=\partial_\nu V_\mu{}^i-\partial_\mu V_\nu{}^i$. The f^{ijk} are proportional to the structure constants of the group SU_3.[3] The corresponding term with d^{ijk} (the totally symmetric symbol) is forbidden because of Bose statistics.

Generalizing the current of Eq. (3) we have the unitary axial vector,

$$K_\mu{}^i=\Lambda(\partial_\mu P^i+2hf^{ijk}V_\mu{}^jP^k)\,. \tag{6}$$

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[1] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).
[2] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960).
[3] Our notation is: Greek indices run from 0 to 3. $a\cdot b=a_\mu b_\mu=a_0b_0-\mathbf{a}\cdot\mathbf{b}$. Latin indices except in Eqs. (1), (2), and (3) run from 1 to 8. The unitary symmetry notation is that of M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).